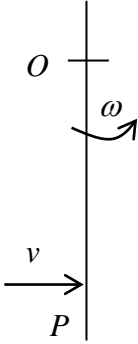


Mock Paper Mark Scheme

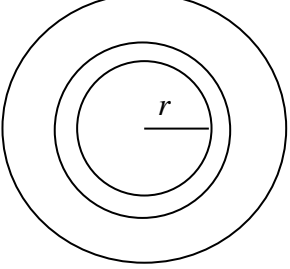
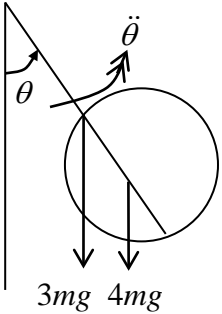
Advanced Subsidiary/Advanced GCE General Certificate of Education

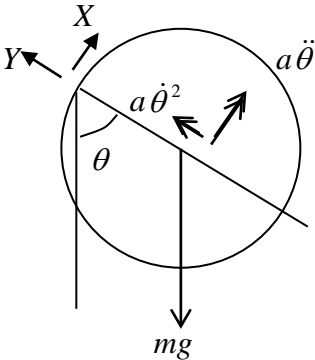
Subject **MECHANICS**

Paper No. **Mock M5**

Question number	Scheme	Marks
<p>1. (a)</p> <p>(b)</p>	<p>Reaction R is perpendicular to the wire, since the wire is smooth. Hence $\mathbf{R} \cdot \mathbf{d} = 0$.</p> <p>$\overrightarrow{AB} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) - (\mathbf{i} + \mathbf{j} - 3\mathbf{k}) = 2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$</p> <p>Work energy: $[2\mathbf{i} + 3\mathbf{j} + (x - 4.9)\mathbf{k}] \cdot (2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) = \frac{1}{2} \times 0.5(4^2 - 2^2)$</p> <p>$4 - 6 + 5(x - 4.9) = 3 \Rightarrow x = 5.9$</p>	<p>B1 (1)</p> <p>M1</p> <p>M1 A1 A1</p> <p>A1 (6)</p> <p style="text-align: right;">(7)</p>
2.	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">  </div> <div> <p>MI of rod about O is $\frac{1}{3}m(2a)^2 + ma^2 = \frac{7}{3}ma^2$</p> <p>Moment of momentum: $3mv \times x = (\frac{7}{3}ma^2 + mx^2)\omega$</p> <p>$x\omega = \frac{2}{3}v$</p> <p>$\Rightarrow 3m \times \frac{3x\omega}{2} \times x = (\frac{7}{3}ma^2 + mx^2)\omega$</p> <p>$\Rightarrow \frac{7}{2}x^2 = \frac{7}{3}a^2 \Rightarrow x = a\sqrt{\frac{2}{3}}$</p> </div> </div>	<p>B1</p> <p>M1 A1 A1 ft</p> <p>M1</p> <p>M1 A1 (7)</p>
3.	<p>Auxiliary equation $m^2 + 4m + 3 = 0 \Rightarrow m = -1$ or -3</p> <p>$\Rightarrow \mathbf{r} = \mathbf{A}e^{-t} + \mathbf{B}e^{-3t}$</p> <p>$t = 0, \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow \mathbf{A} + \mathbf{B} = 2\mathbf{i}$</p> <p>$t = 0, \dot{\mathbf{r}} = 2\mathbf{j} \Rightarrow -\mathbf{A} - 3\mathbf{B} = 2\mathbf{j}$</p> <p>Solving: $-2\mathbf{B} = 2\mathbf{i} + 2\mathbf{j}$</p> <p>$\Rightarrow \mathbf{B} = -(\mathbf{i} + \mathbf{j})$</p> <p>$\mathbf{A} = (3\mathbf{i} + \mathbf{j})$</p> <p>$\mathbf{r} = (3\mathbf{i} + \mathbf{j})e^{-t} - (\mathbf{i} + \mathbf{j})e^{-3t}$</p> <p>$t = \ln 2 \Rightarrow e^{-t} = \frac{1}{2}, e^{-3t} = \frac{1}{8}$</p> <p>$\mathbf{r} = \frac{1}{2}(3\mathbf{i} + \mathbf{j}) - \frac{1}{8}(\mathbf{i} + \mathbf{j}) = \frac{11}{8}\mathbf{i} + \frac{3}{8}\mathbf{j}$</p>	<p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1 (10)</p>

Question number	Scheme	Marks
<p>4. (a) $\mathbf{F} = (2\mathbf{j} + 3\mathbf{k}) + (\mathbf{i} + 4\mathbf{k}) = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ $\mathbf{F} = \sqrt{(1 + 4 + 49)} = \sqrt{54} \text{ N}$</p> <p>(b) \mathbf{F} acts through point with p.v. \mathbf{r}</p> $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ $\begin{pmatrix} 7y - 2z \\ z - 7x \\ 2x - y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \\ -2 \end{pmatrix}, = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ <p>so $7y - 2z = 6$, $z - 7x = -3$, $2x - y = 0$ $\Rightarrow y = 2x \Rightarrow$ e.g. $x = 0$, $y = 0$, $z = -3$ Hence equation of line of action of \mathbf{F} is</p> $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$	<p>M1 M1 A1 (3)</p> <p>M1</p> <p>A1 A1, A1</p> <p>M1 A1</p> <p>M1 A1 (8)</p>	<p>(11)</p>
<p>5.</p> <p>$\xrightarrow{\quad} v$ (Time δt) $\xrightarrow{\quad} v + \delta v$ $\xrightarrow{\quad} v + c$</p> <p>$\underbrace{\hspace{1.5cm}}_m$ $\underbrace{\hspace{1.5cm}}_{m + \delta m}$ $\underbrace{\hspace{1.5cm}}_{-\delta m}$</p> <p>$(m + \delta m)(v + \delta v) + (-\delta m)(v + c) - mv = 0$ $\Rightarrow m dv - c dm = 0$</p> <p>$\int_v^{\frac{2}{3}v} dv = c \int_M^m \frac{dm}{m}$</p> <p>$\left[v \right]_v^{\frac{2}{3}v} = c \left[\ln m \right]_M^m$</p> <p>$-\frac{1}{3}v = c \ln \left(\frac{m}{M} \right)$</p> <p>$\Rightarrow m = Me^{-\frac{v}{3c}}$</p> <p>$\therefore$ Fuel used = $M - m = M(1 - e^{-\frac{v}{3c}})$</p>	<p>reducing equation</p> <p>separating variables</p> <p>integration</p> <p>applying limits</p> <p>eliminating ln and $m =$</p>	<p>M1 A2, 1, 0</p> <p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>M1 A1 (11)</p>

Question number	Scheme	Marks
6. (a)	 <p>MI of element = $2\pi r \delta r \times r^2$ $m = \pi \rho a^2$ $\Rightarrow I = \frac{2m}{a^2} \int_0^a r^3 dr$ $= \frac{2m}{a^2} \left[\frac{r^4}{4} \right]_0^a = \frac{1}{2} ma^2$</p>	M1 M1 A1 (3)
(b)	$I = I_{\text{rod}} + I_{\text{disc}} = \frac{4}{3} \times 3m \times a^2 + \frac{1}{2} \times 4m \left(\frac{a}{2} \right)^2 + 4m \left(\frac{3a}{2} \right)^2$ $= 4ma^2 + \frac{ma^2}{2} + 9ma^2$ $= \frac{27}{2} ma^2$	B1, M1 A1 (3)
(c)	 $\frac{27}{2} ma^2 \ddot{\theta} = -3mg \times a \sin \theta - 4mg \times \frac{3a}{2} \sin \theta$ $= -9mga \sin \theta$ $\ddot{\theta} = -\frac{2g}{3a} \sin \theta$ <p>Small oscillations $\Rightarrow \sin \theta \approx \theta$</p> $\Rightarrow \ddot{\theta} = -\frac{2g}{3a} \theta \text{ which is SHM}$ $T = 2\pi \sqrt{\frac{3a}{2g}}$	M1 A2, 1, 0 M1 A1 A1 (6) (12)

Question number	Scheme	Marks
7. (a)	<p>MI of sphere about $L = \frac{2}{5}ma^2 + ma^2 = \frac{7}{5}ma^2$</p> <p>Energy: $\frac{1}{2} \times \frac{7}{5}ma^2 \times \frac{18g}{7a} - \frac{1}{2} \times \frac{7}{5}ma^2 \times \omega^2 = mga(1 - \cos \theta)$</p> <p>$\Rightarrow \frac{7}{10}a\omega^2 = \frac{8}{10}g + g \cos \theta$</p> <p>$\omega^2 = \frac{g}{7a}(8 + 10 \cos \theta) = \frac{2g}{7a}(4 + 5 \cos \theta)$</p>	B1 M1 A2, 1, 0 A1 (5)
7. (b)	<p>$\frac{7}{5}ma^2\ddot{\theta} = -mga \sin \theta \Rightarrow \ddot{\theta} = -\frac{5g}{7a} \sin \theta$</p> <p>[or $2\omega\dot{\omega} = -\frac{10g}{7a} \sin \theta \times \omega \Rightarrow \dot{\omega} = -\frac{5g}{7a} \sin \theta$]</p>	M1 A1 (2)
7. (c)	 <p>$\dot{\theta} = 0$ when $\cos \theta = -\frac{4}{5}$</p> <p>$Y - mg \cos \theta = ma\dot{\theta}^2$</p> <p>$\dot{\theta} = 0, \cos \theta = -\frac{4}{5} \Rightarrow Y = -\frac{4mg}{5}$</p> <p>$X - mg \sin \theta = ma\ddot{\theta}$</p> <p>$\dot{\theta} = 0$ and $\cos \theta = -\frac{4}{5} \Rightarrow \sin \theta = \frac{3}{5}$</p> <p>$\Rightarrow \ddot{\theta} = -\frac{3g}{7a}$</p> <p>$\Rightarrow X = \frac{3mg}{5} - \frac{3mg}{7} = \frac{6mg}{35}$</p> <p>Magnitude of force $= \sqrt{(X^2 + Y^2)}$</p> <p>$= mg \left[\left(\frac{6}{35}\right)^2 + \left(\frac{4}{5}\right)^2 \right]^{\frac{1}{2}}$</p> <p>$\approx 0.818 mg$</p>	B1 M1 M1 A1 M1 M1 A1 M1 A1 (10) (17)